Zamiast notatek rozdzial z

Stellar astrophysical fluid dynamics (Cambridge University Press, 2003)

1

On the diversity of stellar pulsations

WOJCIECH A. DZIEMBOWSKI

Warsaw University Observatory and
Copernicus Astronomical Centre, Polish Academy of Sciences, Warsaw, Poland

Pulsation is a common phenomenon in stars. It occurs in a wide range of their masses and in all evolutionary phases, exhibiting large variety of forms. Stochastic driving and just two distinct instability mechanisms are the cause of the widespread phenomenon. The diversity of pulsation properties in stars across the H-R diagram is partially explained in terms of differences in the ranges of unstable modes and in terms nonlinear mechanisms of amplitude limitation. Still a great deal remains to be explained.

1.1 Introduction

Excitation of the fundamental radial mode was the essence of the pulsation hypothesis when it was first proposed by Ritter in 1879, as an explanation of periodic variability in stars. Radial symmetry of the motion was confirmed for a number of objects by means of observational tests. Excitation of the same, presumably fundamental, mode in all δ Cephei type stars got support in the discovery of the period-luminosity relation, which at some point seemed unique. Soon, the hypothesis that only the fundamental radial mode may be excited became a dogma like the earlier one that stars do not vary.

Referring to Schwarzschild's (1942) suggestion that RRc stars might be first overtone pulsators, Rosseland (1949) wrote: *This hypothesis involves the very difficult problem of how to excite a higher mode to pulsation while leaving the fundamental mode unexcited.* Referring to Ledoux's (1951) proposal that nonradial modes are excited in β Canis Majoris stars, Chandrasekhar and Lebovitz (1962), though not questioning the claim, still had this comment: *... one is generally reluctant to accept suggestions to appeal* directly *to the excitation of non-radial modes (besides the radial modes) on the grounds that such modes should be highly damped relative to radial modes* *and, further, that their excitation would be "difficult" in view of the possible source of such excitation being deep in the interior.*

Today we know that in many stars nonradial modes are excited by the same mechanism as radial ones. In many others only the former are unstable. Firm evidence for overtone pulsation was found among most classical pulsators such as Cepheids and RR Lyrae stars. In agreement with theoretical predictions, even second pulsators have been identified. The latter finding is relatively new. It came as a by-product of massive photometric surveys aimed at the detection of microlensing events.

Astronomers were aware of diversity in the form of stellar pulsation from the very beginning of astrophysics. Baily introduced his division of RR Lyrae stars into subtypes a, b, and c already in 1899. Eight years later Blazkho discovered amplitude variations in one of the RRa stars. Not long after, Hertzsprung described variations of the shape of light curves with period in Cepheids. With the progress in observational methods we have learned about a much larger variety of stellar pulsation. We only partially understand how it comes about. Remarkably, we still do not have a fully satisfactory model for the effect discovered by Blazkho.

1.2 Types of stellar pulsation

One natural division of pulsating stars is into *stochastically driven pulsators* and *unstable-mode pulsators*. There are only few distant stars for which we have information about stochastically excited modes. The pattern of mode excitation is the same as in the sun. For the rest of present review I will be concerned only with the latter type and I will subdivide it into *giant-type* and *dwarf-type*.

1.2.1 Giant-type pulsators

It is only after data from massive photometric surveys became available that we have a fair statistics of pulsational behaviour in classical pulsating stars. Table 1.1, which is based on data from Udalski et al. (1999) and Udalski et al. (2000), gives the percentage of various types of pulsating Cepheids in, respectively, the Small and Large Magellanic Clouds as determined from the OGLE II project. We see that the fundamental mode is the most frequent choice but the first-overtone pulsators are common too. Few pure second-overtone Cepheids are found and only in the SMC. Also double-mode pulsators are very rare. These facts call for an interpretation.

No firm evidence as yet has been found for nonradial modes in Cepheid

Table 1.1. *Magellanic Cloud Cepheids pulsating in various modes*

modes	SMC (%)	LMC(%)
fundamental	58.5	56.9
fund. + first ov.	1.2	1.4
first overtone	35.9	37.5
second + first ov.	3.6	4.2
second overtone	0.7	0

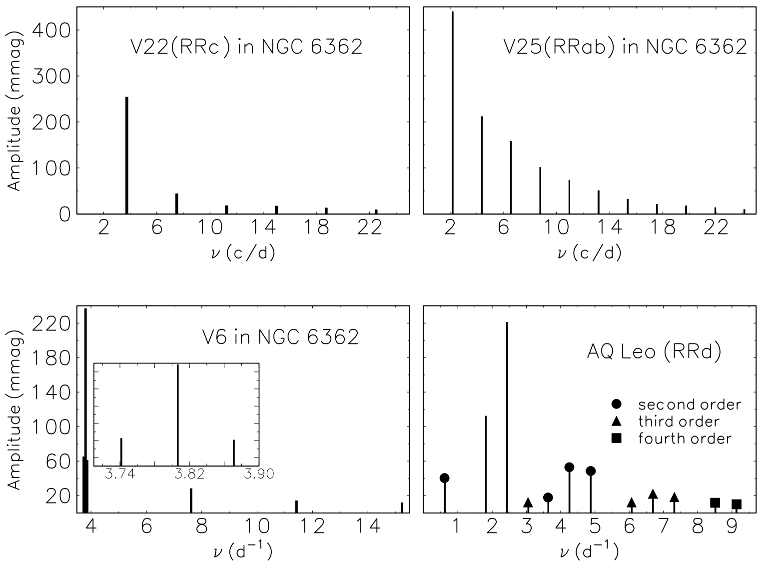

Fig. 1.1. Oscillation spectra for RR Lyrae stars. Data for the NGC 6362 stars are from Olech et al. (2001) and those for AQ Leonis are from Jerzykiewicz & Wentzel (1977). The two stars in the upper row are monomode pulsators. All peaks shown there are at multiples of the pulsation frequencies, which is that of the first overtone for V22 and the fundamental mode for V25. For V6 we see two close side peaks causing Blazkho-type amplitude modulation. In AQ Leonis both the fundamental and first overtone of radial pulsation are excited. The remaining peaks are at various combinations of the two basic frequencies.

pulsation. However, a few cases of long-time amplitude and phase changes were found and remain unexplained. In contrast, long-time modulations are rather common among RR Lyrae stars. The first evidence for nonradial mode excitation in RR Lyrae was found by Olech et al. (1999). The evidence was based on frequency analysis of RR Lyrae light curves, which has revealed the presence of closely spaced peaks. Subsequent analyses (see Kovács, 2002 for a summary) performed on large samples of light curves added many new objects with the same property.

Still the majority of RR Lyrae stars are apparently monoperiodic and pulsating in the fundamental mode (RRab) or first overtone (RRc). The upper panels of Figure 1.1 show examples of oscillation spectra for objects of these two subtypes. The lower panels show the spectra of two types of multiperiodic pulsation. In the left panel we see three closely and equally spaced peaks, which certainly cannot be attributed to different radial modes. The right panel shows the spectrum for AQ Leonis – the first discovered RRd star, which is the adopted designation for objects with the fundamental and first overtone simultaneously present.

Actually, V6 in NGC 6362 is not a strong case for nonradial mode excitation. Although, as we shall see later, its spectrum may be explained in such terms it may also be interpreted in terms of a single radial mode with periodically modulated amplitude. The strong case from the same cluster is the object V37, which has only two close peaks. The observed modulation is then the result of two-frequency beating like in RRd stars but with a longer period. According to surveys summarized by Kovács (2002) the cases of two close peaks are more common.

1.3 Dwarf - type pulsators

Along the main-sequence band there is only one star, BW Vul, that mimic the behaviour of Cepheid and RR Lyrae stars in its pulsation form. It is monoperiodic and of high amplitude, 0.3 mag in the V-band. All remaining pulsators have amplitudes of individual modes below 0.1 mag. Typically, more than one mode is detected if observations are carried out for a long time. A good example is FG Virginis, a δ Scuti type star whose oscillation spectrum is shown in Figure 1.2.

Stars of this type lie in the low-luminosity extension of the Cepheid instability strip. High-amplitude pulsators are found in this type but all lie above the main-sequence band. There is a clear correlation between the pulsation form and the evolutionary status.

Along the main-sequence band, both below and above δ Scuti stars, there are pulsating stars showing striking diversity in the radial orders n of the excited modes. In δ Scuti stars we find p-modes of orders from $n = 1$ up to 7 and some low-order g-modes. Magnetic stars occupying part of the δ Scuti domain choose p-modes of much higher orders ($n > 20$). Immediately below there is a domain of γ Doradus star, which are high-order g-mode pulsators. Above δ Scuti stars, after short break around spectral type A0

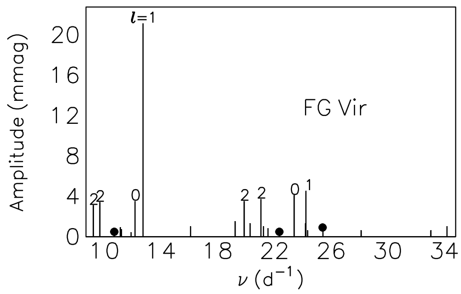

Fig. 1.2. Oscillation spectrum for FG Virginis. Amplitudes and frequencies are from Breger et al. (1998). Numbers on top of the bars indicate the spherical harmonic orders of modes as determined by Viskum et al. (1998). The three bars marked with filled circles are at nonlinear combination frequencies. The remaining 22 bars correspond to eigenmode frequencies

we have SPB stars, which also choose high-order g-modes. At still higher luminosity is the β Cephei domain, where stars pulsate again in low-order p- and g-modes.

There are three domains of g-mode pulsation along the white dwarf cooling sequence and a domain in the hot extension of the horizontal branch of sdB stars which show a similar mode preference as δ Scuti stars. They all, like the main-sequence stars, are multimode low-amplitude pulsators.

1.4 Inference from linear theory

Linear stability calculations for stellar models predict simultaneous instability of a large number of modes leaving to speculation the problem of the final amplitude outcome of the instability. Nonetheless, such calculations yield an important step towards understanding stellar pulsation. Their results that may be directly compared with observations are the ranges in frequency and spherical-harmonic degree ℓ of unstable modes. The agreement is a support for the model in which the driving effect may then be easily identified. On such grounds, we may claim that we understand the origin of oscillations in nearly all types of objects mentioned in the previous section. It is remarkable that, despite the whole richness in the pulsational behaviours, there are only two driving mechanisms that seem to account for all the cases.

We now have a satisfactory interpretation in terms of the opacity mechanism for the two large instability domains in the H-R diagram: the classical Cepheid instability strip and the newly explained B star instability strip. In the first case the driving effect arises in the hydrogen and helium ionization layers. In the second case it arises in a local maximum of opacity caused by iron lines at a temperature of about 2×10^5 K. Finding this new instability strip, which includes main-sequence and subdwarf B stars, followed improvement in stellar opacities. In fact, finding instability in models of B subdwarf models preceded the discovery of oscillations in these stars (Charpinet et al., 1996).

Even at the linear theory level there are unsolved problems. They concern role of convection, which is far more important for stars of the Cepheid instability strip, especially but not only in determining its red edge. The pioneering efforts by Gough (1977) to include the effects of turbulent convection in stellar pulsation were followed by many. Nonetheless an accurate and credible modeling is missing. In consequence, our understanding of variability in stars cooler than Cepheids is the poorest. In many cases we are not even sure whether the variability is due to pulsation and, if it is, whether it is due to unstable or to stochastically driven modes.

Convection does not always exert a damping effect on oscillations. Brickhill (1983) first noted that in the case of slow modes, modulation of the convective flux during the pulsation cycle promotes driving. He proposed this effect as the cause of g-mode excitation in ZZ Ceti stars – oscillating white dwarfs. There are recent developments of this idea by Goldreich & Wu (1999). This driving mechanism is less common in stars than the classical opacity-mechanism but it is the only alternative mechanism leading to mode instability that may be associated with the observed stellar variability. In addition to ZZ Ceti stars the mechanism may work in oscillating DB (helium) white dwarfs and possibly in γ Doradus stars (Wu, 2002).

Typically, in both giant- and dwarf-type pulsators models, there is a large number of unstable modes. Figures 1.3 and 1.4 show the driving rates as a function of frequency for low-degree modes in representative models of δ Scuti and RR Lyrae stars. The former, which approximately fits data on FG Vir (see Figure 1.2), describes a star in an advanced main-sequence evolutionary phase. The latter is a model of a horizontal-branch star in the advanced core helium burning phase. Let us note the differences. The frequency range of the unstable modes is significantly wider in the main-sequence star. The instability of radial modes extends in this case from radial order $n = 1$ to 7. The wide range of mode frequencies is indeed observed in δ Scuti stars. Our selected model reproduces very well the frequency range of modes detected in FG Vir. In helium burning pulsators, instability goes at most up to $n = 3$ and this is the highest-order mode detected in such stars.

More important differences are seen in the nonradial mode properties.

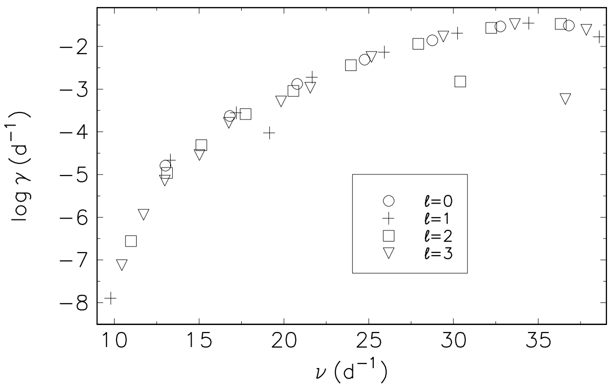

Fig. 1.3. Growth rates and frequencies of low-degree ($\ell \leq 3$) modes in a model of a δ Scuti star. The mass is 1.8 times solar, the initial composition is similar to that of the sun and the evolutionary status is an advanced phase of hydrogen burning in the convective core (the central abundance $X_{\rm c}$ by mass of hydrogen is reduced to 0.085)

.

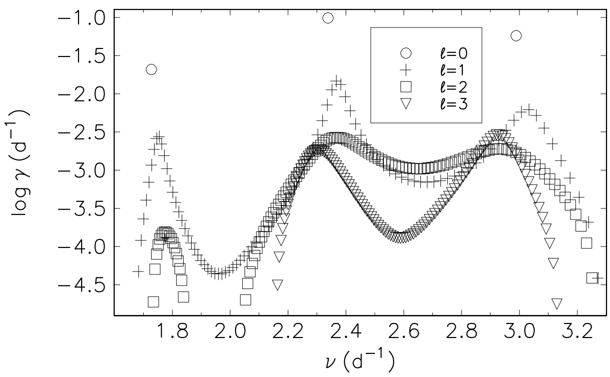

Fig. 1.4. Growth rates and frequencies of low-degree ($\ell \leq 3$) modes in a model of a RR Lyrae star. The mass is 0.67 times solar, the initial composition is typical for population II objects and the evolutionary status is an advanced phase of helium burning in the convective core (the helium abundance is reduced to 17% of the original value). Lack of symbols corresponding to $\ell = 2$ and 3 in certain frequency ranges means lack of unstable modes.

There are many more nonradial modes between consecutive radial modes in the RR Lyrae model than in the δ Scuti model. Spectra of nonradial modes are actually denser than shown in Figures 1.3 and 1.4, as each of the modes is split into $2\ell+1$ components by rotation. The much greater mode density in the RR Lyrae model is a consequence of the much larger values

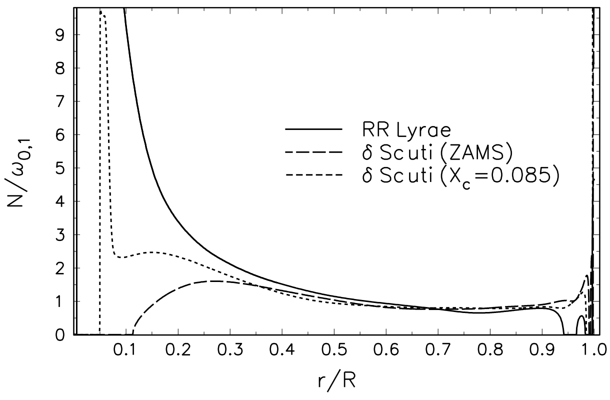

Fig. 1.5. The ratio of the Brunt-Väisälä to the fundamental radial mode frequency in the models used in Figures. 1.3, 1.4 and in a model ZAMS star. In the ZAMS model the central hydrogen abundance $X_{\rm c} = 0.7$. The maximum value of the ratio in the RR Lyrae model is about 300 and it is reached at $r/R = 0.007$

of the Brunt-Väisälä frequency N in the interior of this star. In Figure 1.5 we see the behaviour of $N(r)$ in the two models discussed and in a ZAMS star of the same mass as our selected δ Scuti star. The effect of evolution is a growth of N in the interior and a development of a gravity wave (G) cavity there.

The radial order n_g associated with the G cavity is approximately given by

$$n_g \approx \frac{\sqrt{\ell(\ell+1)}}{\pi} \int \sqrt{\left(\frac{N}{\omega}\right)^2 - 1}\frac{\mathrm{d}r}{r} \tag{1.1}$$

(e.g. Van Hoolst et al., 1998), where $\omega = 2\pi\nu$ is the angular frequency of the mode. The integral should be taken over the G cavity. The frequency distance between consecutive modes of the same degree is thus estimated as

$$\Delta_{\ell,n_g} \approx \frac{\omega}{n_g} \, . \tag{1.2}$$

In the RR Lyrae star model, at $\ell = 1$ and a frequency corresponding to the fundamental radial mode, we get $n_g = 180$.

Already in the evolved main-sequence star, some additional nonradial modes result from the growth of the G cavity around the shrinking convective core. Also in this model we see structures in the $\gamma(\nu)$ dependence reflecting mode trapping effect. Local minima correspond to modes partially trapped in the G cavity. The effect is much more dramatic in the RR Lyrae star. For all nonradial modes most of the oscillation energy is confined to the

G cavity. The relative contribution from the acoustic cavity is only about 10% for the $\ell = 1$ modes corresponding to local maxima. At this degree still most of contribution to damping and driving arises in the outer layers so that the difference between in driving rates between the $\ell = 0$ and $\ell = 1$ modes is mainly due to the difference in inertia. At $\ell = 2$ and 3, damping in the G cavity results in mode stability in certain frequency ranges.

Even for the most strongly trapped nonradial modes, the driving rates, γ, are significantly lower than those for the closest radial modes, which is not true in the δ Scuti star. We may be tempted to take this fact as the explanation why RR Lyrae and other evolved stars exhibit preference for radial pulsation. However, more detailed comparison with observations warns us against such inference. The modes detected in FG Virginis at $\nu \approx 10\mathrm{d}^{-1}$ have their driving rates lower by six orders of magnitude than those at $\nu \approx 30\mathrm{d}^{-1}$. The dependence of amplitude on frequency shown in Figure 1.2 bears no resemblance to the $\gamma(\nu)$ dependence shown in Figure 1.3. Clearly, we cannot rely on the driving rates for predicting amplitudes of modes surviving in the nonlinear development.

1.5 Saturation of the linear instability

Christy (1964) was the first to construct fully nonlinear models of Cepheids and RR Lyrae stars. His models converged to a periodic constant amplitude pulsation state, which – according to his interpretation – was reached through a saturation, that is, through modification induced by pulsation in the mean structure, leading to zeroing driving rate. Depending on mean value of L and T_{eff}, the terminal state was either fundamental or first-overtone pulsation. Later on Stobie (1969) found also second-overtone pulsation in his Cepheid models. It took 30 years to find such a form of pulsation in real objects. Stellingwerf (1975) with his novel method was able to determine exclusive domains in the H-R diagram of first overtone and fundamental mode pulsation and an intermediate domain, which he named the EO (either-or), where both modes were possible depending which one was excited first.

The origin of double-mode pulsation was not understood for a long time and even now the problem is not fully clarified. It has been approached in a number of works by means of numerical solution of the full nonlinear problem and with the amplitude equation formalism. Nonlinear saturation shows up at the cubic order in pulsation amplitudes. With our cubic order formalism we (Dziembowski & Kovács, 1984) derived a simple criterion for double-mode pulsation. Here I outline our analysis.

The nonlinear driving rates were written in the form

$$\gamma_{j,N} = \gamma_j \left(1 + \sum_k \alpha_{jk} A_k^2\right) . \tag{1.3}$$

Only the case of two linearly unstable modes was considered. It was assumed that all saturation coefficients α_{jk} are negative, which is necessary for saturation and confirmed by subsequent numerical calculations. The terminal amplitudes are determined by the set of two equations (1.3) with $\gamma_{j,N} = 0$. For monomode solutions, which with our assumptions always exist, we have

$$A_j^2 = -\frac{1}{\alpha_{jj}} . \tag{1.4}$$

It is stable if $s_k \equiv \alpha_{kj}/\alpha_{jj} - 1 > 0$, where $k \neq j$, which means that the mode is more effectively saturating instability of the competing mode than that of its own. If $s_1 > 0$ and $s_2 > 0$ then we are in the EO domain. The double-mode solution

$$A_1^2 = -\frac{1}{\alpha_{11}} \frac{s_1}{s_1 + s_2 - s_1 s_2}, \quad A_2^2 = -\frac{1}{\alpha_{22}} \frac{s_2}{s_1 + s_2 - s_1 s_2}$$

exists if $s_1 s_2 > 0$. However, it is stable only if $s_1 < 0$ and $s_2 < 0$. Thus, monomode and double-mode pulsations are mutually exclusive.

The fact that that double-mode pulsations are so rare may be interpreted in two ways. Either the range of parameters leading to $s_1 < 0$ and $s_2 < 0$ is very narrow or we have $s_1 > 0$ and $s_2 > 0$ but the amplitude of mode 1 cannot reach its saturation value given in equation (1.4) due to an accidental resonance with a damped mode. We preferred the second way and suggested that there is a 2:1 resonance with a higher-order radial mode. The problem with this idea, which was realized later, was lack of required resonance in realistic models of double-mode pulsators.

Successful numerical simulations of double-mode pulsation in Cepheids (Kolláth et al., 1998) and RR Lyrae stars (Feuchtinger, 1998) were obtained only after effects of convection were included. The resonance played no role in this models. In a recent paper Kolláth et al. (2002) interpreted these results with the amplitude equations. They found that the condition $s_1 < 0$ and $s_2 < 0$ was satisfied in their double-mode pulsators. This was never the case in purely radiative models. Unfortunately, they did not identify the specific effect of convection responsible for the enhancement of self-saturation, causing stabilization of the double-mode pulsation.

An unpleasant aspect of this solution is the fact that it rests on a crude description of convection in which there are four adjustable parameters. Furthermore, there is a problem of nonradial modes, whose presence has been ignored in all numerical models so far. We will see later that a resonant coupling with those modes may play a role for the properties of radial pulsation. There is also a question regarding the role of strongly unstable high-degree modes. Many such modes, having driving rates similar to the radial modes, exist in all models of RR Lyrae star and Cepheids (e.g. Van Hoolst et al., 1998). If the saturation is the dominant amplitude-limiting effect, then we should expect that often one of such modes wins the competition. The resulting pulsation would be undetectable by means of photometry. Observations do not indicate that it may be the case. The RR Lyrae strip seems filled up with the pulsators. Thus, we have to admit that we still do not understand why radial monomode pulsation is preferred by stars in the upper Cepheid instability strip.

1.6 Amplitude limitation by resonances

That resonances may play a role in stellar pulsation has been realized well before numerical modeling became possible. This is what Rosseland (1949) wrote about about early attempts to explain the shapes of Cepheid light curves: *The conclusion seems unavoidable that some particular property of the star plays an active role in shaping the curves. It may be something like a resonance between the fundamental and higher mode suggested by Woltjer (1935, 1937).*

Modern investigations have confirmed this.

1.6.1 The 2:1 resonance

The Hertzsprung sequence of Cepheid light curves has been successfully explained in terms of the frequency distance from the 2:1 resonance centre between the fundamental mode ($n = 1$) and the second overtone ($n = 3$) (Simon & Schmidt, 1976, Buchler et al., 1990).

Also Woltjer was the first to point out that the 2:1 resonance causes amplitude limitation. Since it is a lower-order effect in terms of amplitude, one might expect that it should be more efficient in amplitude limitation than saturation. However, it does not seem to be the case in Cepheids. The amplitudes at 10-day period, which is the resonance centre, are not markedly lowered. The point is that this resonance may be a sole amplitude-limiting effect only if the damping of modes of higher radial order is fast enough. A stable double-mode solution exists only if $2\gamma_1 + \gamma_3 < 0$.

A critical role for the 2:1 resonance in the amplitude limitation was found in limiting the growth of the ϵ-mechanism-driven instability of high-mass stars by Papaloizou (1973). He showed that the resonance between the unstable fundamental mode and the stable first overtone is very effective preventing a catastrophic mass loss, which was suggested by earlier investigators.

1.6.2 Parametric resonance and dwarf and giant dichotomy

Three-mode coupling caused by the parametric resonance is another lowest-order nonlinear effect leading to amplitude limitation. In this case the effect is due to dissipation of energy by a pair of linearly stable (daughter) modes for whose sum are close to frequency of an unstable (parent) mode. Denoting with subscripts a and b the daughter modes and with c the parent mode we have

$$\omega_c = \omega_b + \omega_a + \Delta\omega,$$

with $|\Delta\omega| \ll \omega_c$ and

$$\gamma_c > 0, \quad \gamma_a < 0, \quad \gamma_b < 0.$$

An exponential growth of modes a and b occurs if the amplitude of mode c exceeds the critical value, which (e.g. Vandakurov, 1981) is given by

$$A_{c,\mathrm{crit}} = \sqrt{\frac{\gamma_a\gamma_b}{C_{abc}}\left[1 + \left(\frac{\Delta\omega}{\gamma_d}\right)^2\right]}\,, \tag{1.5}$$

where $\gamma_d = \gamma_a + \gamma_b$. The coupling coefficient C_{abc} is a volume integral with integrand containing products of eigenfunctions of the three involved modes. The general expression is complicated (Dziembowski, 1982) but it is easy to show that $C_{abc} \neq 0$ only if the azimuthal orders satisfy the condition $m_c = m_a + m_b$ and the difference between the two highest ℓ is not larger than the lowest one. For instance, if the parent mode is radial then the daughter modes must have $m_a = -m_b$ and $\ell_a = \ell_b$.

Freedom in choosing ℓ_a and m_a allows fine frequency tuning. The frequency distance, Δ_{ℓ,n_g}, decreases approximately as ℓ^{-1} (see equations 1.1 and 1.2), hence considering daughter modes with $\ell \to \infty$ we may approach $\Delta\omega = 0$. This favours high-ℓ mode excitation. The opposite effect is that of damping. If the quasi-adiabatic approximation applies then we have approximately (e.g. Van Hoolst et al., 1998)

$$\gamma \approx \frac{\bar{N}^2}{\tau_g}\frac{\ell^2}{\omega^2}\,, \tag{1.6}$$

where τ_g is the thermal time scale of the G cavity. In main-sequence stars the instability first appears at certain intermediate though still rather high ℓ values implying that the daughter modes are most likely undetectable. Important observable consequences occur for the parent mode, whose amplitude may be reduced to the level not much exceeding $A_{c,\mathrm{crit}}$ and may be modulated.

The character of the terminal pulsation state resulting from the interaction between the parent and the daughter modes depends on the mismatch, $\Delta\omega$, and the driving (damping) rates γ. Stable stationary solutions with the parent-mode amplitude given by the right-hand side of equation (1.5), but with γ_d replaced by $\gamma_s = \gamma_c + \gamma_d$ exist in wide range of parameters (see Wersinger et al., 1980, Dziembowski, 1982). Outside that range, in particular for a close resonance, only time-dependent amplitude limitation is possible. The solution may take a form of a single- or multi-periodic limit cycle and, going through a series of period doubling, become chaotic. Still equation (1.5) may be used for a crude estimate of the mean amplitude. If $\gamma_s < 0$ then amplitude limitation in any form by the sole effect of the parametric resonance is not possible.

My first application of the theory of parametric resonance was to estimate the amplitude of an $\ell = 1$ g-mode in the sun. Dilke & Gough (1972) showed that the mode may be driven by the ϵ mechanism and speculated that it might reach high amplitude, high enough to mix the solar interior. This was an ingenious idea invented to solve the neutrino deficit problem. The results of my calculations (Dziembowski, 1983) were unfortunately discouraging. The parametric instability was found to set in at very low amplitudes, far lower than needed for mixing. The next application was to explain the low pulsation amplitudes of δ Scuti stars (Dziembowski & Królikowska, 1985). We found that the amplitudes of unstable modes were limited by the three-mode interaction to the level of 1 - 10 mmag, which was in a rough agreement with observations. The values are well below the ones needed to saturate the instability.

It seemed that we were on the road toward explaining the systematic difference between giant and dwarf pulsators. In Cepheids and RR Lyrae stars the parametric resonance does not prevent high pulsation amplitudes for two reasons. Damping rates of daughter modes are much higher than in δ Scuti stars due to the much higher $\bar{N}$ (see Figure 1.5) and τ_g is shorter. The second reason is a weaker coupling (smaller C_{abc}) between the parent radial and the potential daughter modes. The latter are trapped in the deep interior, where the former ones have very low amplitudes. The truth is, however, that not much happened after those works. The difficulty is that most likely much more than just one pair of daughter modes is excited at the onset of the parent-mode instability. It is easy to show that constant-amplitude solutions do not exist if there are more than two pairs. Beyond that, the problem is difficult and to my best knowledge, was never solved.

1.6.3 Higher-order parametric resonance and the Blazkho effect

Resonant coupling between radial and nonradial modes of similar frequencies is a third-order effect in the amplitude expansion. Nonetheless, it may have a greater impact on RR Lyrae pulsation than the lower-order resonant coupling due to the properties of the mode-trapping pattern. We have seen in Figure 1.4 that nonradial modes with frequencies close to those of radial modes are also close to the maxima of the driving rates. This means relatively large amplitudes in the acoustic cavity, hence stronger coupling to radial modes and lower damping rates. Both effects promote parametric instability. Figure 1.4 also shows that the trapping favours excitation of $\ell = 1$ modes.

Van Hoolst et al. (1998) derived the following expression for the amplitude of the parent mode amplitude at the onset of the instability:

$$A_0^2 > \sqrt{\frac{\Delta\omega^2 + \gamma_{\ell,N}^2}{C_{00\ell\ell}}} \, .$$

It is similar to equation (1.5), but specialized to a radial mode and it takes into account saturation of driving by the radial mode, which makes $\gamma_{\ell,N}$ negative. Our survey (Dziembowski & Cassisi, 1999) of realistic models of RR Lyrae stars has revealed that excitation of radial modes is quite likely. The probability ranges from 0.3 to 0.9. A study of the nonlinear development (Nowakowski & Dziembowski, 2001) has shown that in this case there is always a constant-amplitude pulsation state. If a single $\ell = 1, m = 0$ mode is excited then the effect may easily escape detection. The phases of the two modes are locked so that variability remains monoperiodic with only slightly modified period. The effect on the amplitude is more significant but always the radial component strongly dominates in the light and radial velocity variations.

A more interesting situation arises if a pair of $\ell = 1$ modes is excited. The following possibilities exist: the two modes may still be (i) axisymmetric but belong to different triplets; it may be a $m = \pm 1$ pair of (ii) the same triplet or; (iii) different triplets. What matters is only that the frequency mismatch

$$\Delta\omega = 2\omega_0 - (\omega_{1,k,m_k} + \omega_{1,n,m_n}),$$

where subscripts k and n stand for radial orders of $\ell = 1$ modes while

m_k and m_n are the corresponding azimuthal orders, is small and that the modes are located possibly close to maxima of the linear driving rate. In this case, the phase lock produces an equidistant triplet with the central peak corresponding to the radial mode, giving rise to a phase and/or amplitude modulated pulsation.

The model predicts equal amplitudes of the side peaks reaching up to 0.35 of the central-peak amplitude. It is a viable model for stars like V6 in NGC 6362 and it is appealing because, if correct, then from the Blazkho period we get valuable information about the deep stellar interior. Unfortunately, the model does not explain all RR stars with variable amplitudes. It is certainly not applicable to cases when only two close peaks are detected. It is also not readily applicable if two side peaks are seen but with too large or very unequal amplitudes.

1.7 Final remarks

Today the main emphasis in studies of pulsating star research is put on asteroseismology, that is on using pulsation data to constrain stellar models, as well as on other applications such as determination of distances to stellar systems. The physics of the pulsation phenomenon is often regarded as sufficiently well understood. In this review I have been advocating an opposite view. I presented a number of problems posed by observations where we are lacking physical understanding.

Even though the domains of occurrence of various types of oscillations in the H-R diagram are well reproduced with the results of linear stability analyses, still certain problems within the linear stability theory remain, awaiting progress in the treatment and understanding of the interaction between convection and pulsation. Among the unsolved problems, the outstanding one is the cause of the universal variability in red giants.

Modern observations do not challenge the hypothesis of pure radial monomode pulsation for the majority of Cepheids and RR Lyrae stars. These objects seem indeed to be amazingly simple natural heat engines. Why this simplest form of motion is chosen we do not quite understand. Many other pulsation modes are unstable. Explaining complexity in nature has became fashionable in science. In my view, priority should be given to explaining simplicity. The question why pulsation is so simple was not answered by our predecessors in the field of stellar pulsation research. Also our generation did not give a fully satisfactory answer. I hope that one of our younger colleagues will tell us why.

Acknowledgement: This work was supported in part by the Polish grant KBN 2P03D 030 20.

References

Breger, M. et al., 1998, *A&A*, **331**, 271.
Brickhill, A.J., 1983, *MNRAS*, **204**, 537.
Buchler, J.R., Moskalik, P. & Kovács, G., 1990, *ApJ*, **351**, 617.
Chandrasekhar, S. & Lebovitz, N. R., 1962, *ApJ*, **136**, 1105.
Charpinet, S., Fontaine, G., Brassard, P. & Dorman, B. 1996, *ApJ*, **471**, L103.
Christy, R.N. 1964, *Rev. Mod. Phys.*,, **36**, 555.
Dilke, F.W.W. & Gough, D.O., 1972, *Nature*, **240**, 262.
Dziembowski, W.A., 1982, *Acta Astron.*, **32**, 147.
Dziembowski, W.A., 1983, *Solar Phys.*, **82**, 259.
Dziembowski, W.A. & Cassisi, S., 1999, *Acta Astron.*, **49**, 371.
Dziembowski, W.A. & Królikowska, M., 1985, *Acta Astron.*, **35**, 6.
Dziembowski, W.A. & Kovács, G., 1984, *MNRAS*, **196**, 731.
Feuchtinger, 1998, *A&A*, **337**, L29.
Goldreich, P. & Wu, Y., 1999, *ApJ*, **511**, 904.
Gough, D.O., 1977, *ApJ*, **214**, 196.
Jerzykiewicz, M. & Wentzel, W., 1977, *Acta Astron.*, **27**, 35.
Kolláth, Z., Beulieu, J.P., Buchler, J.R. & Yecko, P., 1998, *ApJ*, **502**, L55.
Kolláth, Z., Buchler, J.R., Szabó, R. & Csubry, Z., 2002, *A&A*, in press.
Kovács, G., 2002, in *IAU Colloquium 185, Radial and Nonradial Pulsations as Probes of Stellar Physics* eds. C. Aerts, T. R. Bedding and J. Christensen-Dalsgaard, (ASP - Conference Series), in press.
Ledoux, P., 1951, *ApJ*, **114**, 373.
Nowakowski, R.M. & Dziembowski, W.A., 2001, *Acta Astron.*, **51**, 5.
Olech, A., et al., 1999, *AJ*, **162**, 442.
Olech, A., et al., 2001, *MNRAS*, **321**, 421.
Papaloizou, J.C.B., 1973, *MNRAS*, **162**, 143.
Rosseland, S., 1949, *The pulsation theory of variable stars*, The Clarendon Press (Oxford).
Schwarzschild, M., 1942, *ApJ*, **94**, 241.
Simon, N.R. & Schmidt, E.G., 1976, *ApJ*, **205**, 162.
Stellingwerf, R.F., 1975, *ApJ*, **195**, 441.
Stobie, R.S., 1969, *MNRAS*, **144**, 511.
Udalski, A., et al., 1999, *Acta Astron.*, **49**, 1.
Udalski, A., et al., 2000, *Acta Astron.*, **50**, 307.
Van Hoolst, T., Dziembowski, W.A. & Kawaler, S.D., 1998, *MNRAS*, **297**, 536.
Vandakurov, Yu.V, 1981, *Soviet Astron. Letters*, **7**, 128.
Viskum, M., et al., 1998, *A&A*, **335**, 549.
Wu, Y.V., 2002, in *IAU Colloquium 185, Radial and Nonradial Pulsations as Probes of Stellar Physics* eds. C. Aerts, T. R. Bedding and J. Christensen-Dalsgaard, (ASP - Conference Series), in press.
Wersinger, J.M., Finn, I.M. & Ott, E. 1980, *Phys. Fluids*, **23**, 1142.