Expressions for light and radial velocity variations due to nonradial pulsations

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In order to obtain theoretical amplitudes and phases due to nonradial pulsations, two inputs are needed

- models of stellar atmospheres,
- nonadiabtic theory of stellar pulsation.

In the case of main-sequence pulsators, it is justified to assume

- plane-parallel and temporary static atmospheres,
- linear pulsations.

To calculate amplitudes and phases of the photometric changes in passband λ, it is most convenient to consider an expression in the complex form (Daszyńska-Daszkiewicz et al. 2002)

$$\mathcal{A}_\lambda(i) = -1.086\varepsilon Y_\ell^m(i,0) b_\ell^\lambda (D_{1,\ell}^\lambda f + D_{2,\ell} + D_{3,\ell}^\lambda) \tag{1}$$

where

$$D_{1,\ell}^\lambda = \frac{1}{4}\frac{\partial \log(\mathcal{F}_\lambda |b_\ell^\lambda|)}{\partial \log T_{\rm eff}}, \tag{2a}$$

$$D_{2,\ell} = (2+\ell)(1-\ell), \tag{2b}$$

$$D_{3,\ell}^\lambda = -\left(2 + \frac{\omega^2 R^3}{GM}\right)\frac{\partial \log(\mathcal{F}_\lambda |b_\ell^\lambda|)}{\partial \log g_{\rm eff}^0} \tag{2c}$$

and i is the inclination angle. The term $D_{1,\ell}^\lambda$ describes the temperature effects, the term $D_{2,\ell}$ stands for the geometrical effects, and the influence of gravity changes is contained in the term $D_{3,\ell}^\lambda$. The terms $D_{1,\ell}^\lambda$ and $D_{3,\ell}^\lambda$ include the perturbation of the limb-darkening, and their ℓ-dependence arises from the nonlinearity of the limb-darkening law.

You find the flux derivatives over effective temperature $\partial \log \mathcal{F}_\lambda / \partial \log T_{\rm eff}$ (atx) and over gravity $\partial \log \mathcal{F}_\lambda / \partial \log g$ (agx), under **atmospheric parameters → flux derivatives**. Here „x" denotes a given photometric passband.

The values of disc-averaging factor b_ℓ^λ, defined as

$$b_\ell^\lambda = \int_0^1 h_\lambda(\mu)\mu P_\ell(\mu) d\mu, \tag{3}$$

are located in **atmospheric parameters** $\rightarrow b_\ell^\lambda, u_\ell^\lambda, v_\ell^\lambda$ **coefficients**. The b_ℓ^λ derivatives $\partial b_\ell^\lambda/\partial \log T_{\rm eff}$ and $\partial b_\ell^\lambda/\partial \log g$ are in **atmospheric parameters** $\rightarrow b_\ell^\lambda, u_\ell^\lambda, v_\ell^\lambda$ **derivatives**. In the case of Kurucz models, the $b_\ell^\lambda, u_\ell^\lambda, v_\ell^\lambda$ values were computed using Claret's (2000) nonlinear formula for the limb-darkening law $h_\lambda(\mu)$. In the case of NeMo models, we relied on the square root law (Barban et al. 2003).

Warning: Because you get derivatives of b_ℓ^λ, and not $\log b_\ell^\lambda$, you have to modify the $D_{1,\ell}^\lambda$ and $D_{1,\ell}^\lambda$ terms, e.g.

$$D_{1,\ell}^\lambda = \frac{1}{4}\left(\frac{\partial \log \mathcal{F}_\lambda}{\partial \log T_{\rm eff}} + \frac{1}{\ln 10 b_\ell^\lambda}\frac{\partial b_\ell^\lambda}{\partial \log T_{\rm eff}}\right)$$

The complex parameter, $f = (f_R, f_I)$, describes the ratio of the flux perturbation to the radial displacement at the level of the photosphere and it is obtained from nonadiabatic calculations. These values you find in **pulsation models** $\rightarrow$ **OPAL (OP)** in the **nadXXX.mrt** flies denoted as **COMPLEX F**.

Then, the amplitudes and phases of the light variation are given by

$$A_\lambda = |\mathcal{A}_\lambda| = \sqrt{(}\mathcal{A}_{\lambda,R}^2 + \mathcal{A}_{\lambda,I}^2), \tag{4}$$

and

$$\varphi_\lambda = \arg(\mathcal{A}_\lambda) = \mathrm{arctg}(\mathcal{A}_I/\mathcal{A}_R), \tag{5}$$

where

$$\mathcal{A}_{\lambda,R} = -1.086\varepsilon Y_\ell^m(i,0) b_\ell^\lambda (D_{1,\ell}^\lambda f_R + D_{2,\ell} + D_{3,\ell}^\lambda),$$
$$\mathcal{A}_{\lambda,I} = -1.086\varepsilon Y_\ell^m(i,0) b_\ell^\lambda D_{1,\ell}^\lambda f_I.$$

You can also calculate the complex bolometric amplitude using the formula of Dziembowski (1977), which in a ,,modern" version reads

$$\mathcal{A}_{\rm bol}(i) = -1.086\varepsilon Y_\ell^m(i,0) b_\ell^{\rm bol}\left[(2+\ell)(1-\ell) + f\right]. \tag{6}$$

The $b_\ell^{\rm bol}$ values you can get from **atmospheric parameters** $\rightarrow b_\ell^\lambda, u_\ell^\lambda, v_\ell^\lambda$ **coefficients**.

The amplitude of the radial velocity variation averaged over stellar disc is obtained from the well-known Dziembowski's (1977) formula

$$\mathcal{A}_{Vrad}(i) = \mathrm{i}\omega R\varepsilon Y_\ell^m(i,0)\left(u_\ell^\lambda + \frac{GM}{R^3\omega^2}v_\ell^\lambda\right), \tag{7}$$

where

$$u_\ell^\lambda = \int_0^1 h_\lambda(\mu)\mu^2 P_\ell(\mu) d\mu, \tag{4a}$$

and

$$v_\ell^\lambda = \ell \int_0^1 h_\lambda(\mu)\mu \left(P_{\ell-1}(\mu) - \mu P_\ell(\mu)\right) d\mu. \tag{4b}$$

The $u_\ell^\lambda, v_\ell^\lambda$ coefficients are another disc-averaging factors which you can download from **atmospheric parameters** $\rightarrow$ $b_\ell^\lambda, u_\ell^\lambda, v_\ell^\lambda$ **coefficients**. From observations, the radial velocity variations are determined by calculating the first moment, $\mathcal{M}_1^\lambda$, of a well isolated spectral line.

References

Barban et al., 2003, A&A, 405, 1095

Claret, A. 2000, A&A, 363, 1081

Castelli F., Kurucz R. L., 2004, the Proceedings of the IAU Symp. No 210, in Modelling of Stellar Atmospheres, eds. N. Piskunov et al. 2003, poster A20

Daszyńska-Daszkiewicz J., Dziembowski W. A., Pamyantykh A. A., Goupil M-J., 2002, A&A 392, 151

Dziembowski W. A., 1977, Acta Astron. 27, 203